Model Equations:

1) $$\frac{dB_C}{dt} = P_n - L_C$$

2) $$\frac{dD_C}{dt} = L_C - R_m - Q_{OC}$$

3) $$\frac{dB_N}{dt} = U_N + U_{Nfix} - L_N$$

4) $$\frac{dD_N}{dt} = L_N + U_{mN} - M_N - Q_{ON}$$

5) $$\frac{dB_P}{dt} = U_P - L_P$$

6) $$\frac{dD_P}{dt} = L_P + U_{mP} - M_P - Q_{OP}$$

7) $$\frac{dE_N}{dt} = I_N + I_{NF} + M_N - U_N - U_{mN} - Q_N$$

8) $$\frac{dE_P}{dt} = I_P + M_P - U_P - U_{mP} - Q_P$$

9) $$B_A = \frac{B_{Amax}\,\gamma_B\,B_C}{B_{Amax} + \gamma_B\,B_C}$$

10) $$B_W = B_C - B_A$$

11) $$P_n = Q_{10Pn}^{(T-15)/10} B_A \frac{P_{max}\,C_a}{k_C + C_a}$$

12) $$U_N = Q_{10UN}^{(T-15)/10} B_A \frac{U_{Nmax}\,E_N}{k_N + E_N}$$

13) $$U_P = Q_{10UP}^{(T-15)/10} B_A \frac{U_{Pmax}\,E_P}{k_P + E_P}$$

14) $$L_C = a_A\,B_A + a_W\,B_W$$

15) $$q_N = \frac{B_C}{B_A/q_{NA} + B_W/q_{NW}}$$

16) $$L_N = \frac{q_N\,B_N}{B_C}\left(a_A\frac{B_A}{q_{LNA}} + a_W\frac{B_W}{q_{LNW}}\right)$$

17) $$q_P = \frac{B_C}{B_A/q_{PA} + B_W/q_{PW}}$$

18) $$L_P = \frac{q_P\,B_P}{B_C}\left(a_A\frac{B_A}{q_{LPA}} + a_W\frac{B_W}{q_{LPW}}\right)$$

19) $$\phi_N = \frac{1}{q_{NW}} + \left(\frac{1}{q_{NA}} - \frac{1}{q_{NW}}\right)\frac{B_A^2}{\gamma_B B_C^2}$$

20) $$\phi_P = \frac{1}{q_{PW}} + \left(\frac{1}{q_{PA}} - \frac{1}{q_{PW}}\right)\frac{B_A^2}{\gamma_B B_C^2}$$

21) $$U_{nC} = P_n - L_C$$

22) $$U_{nP} = U_P - L_P$$

23) $$\psi_N = \min\left(\phi_N U_{nC},\, \frac{\phi_N}{\phi_P} U_{nP}\right)$$

24) $$F_{NP} = \max\left(0, \psi_N - U_N + L_N\right)$$

25) $$U_{Nfix} = Q_{10Nfix}^{(T-15)/10}\,\frac{F_{NF}\,B_A\,F_{NP}}{1 + e^{k_{NF}(B_A - B_{NF})}}$$

26) $$U_{nN} = U_N + U_{Nfix} - L_N$$

27) $$I_{NF} = \gamma_{NF}\,B_W^{2/3}$$

28) $$G_{mC} = Q_{10DC}^{(T-15)/10}\,r_{mC} D_C$$

29) $$G_{mN} = Q_{10DN}^{(T-15)/10} r_{mN} D_N$$

30) $$G_{mP} = Q_{10DP}^{(T-15)/10} r_{mP} D_P$$

31) $$U_{mN} = Q_{10mUN}^{(T-15)/10}\left(\frac{\beta_N\,D_C^2}{q_{mN} D_N}\right)\left(\frac{E_N}{k_{mN} + E_N}\right)$$

32) $$U_{mP} = Q_{10mUP}^{(T-15)/10}\left(\frac{\beta_P\,D_C^2}{q_{mP} D_P}\right)\left(\frac{E_P}{k_{mP} + E_P}\right)$$

33) $$R_m = \max\begin{Bmatrix}(1-\varepsilon_{mC})G_{mC} \\ G_{mC} - q_{mN}(G_{mN} + U_{mN}) \\ G_{mC} - q_{mP}(G_{mP} + U_{mP})\end{Bmatrix}$$

34) $$M_N = G_{mN} + U_{mN} - \frac{(G_{mC} - R_m)}{q_{mN}}$$

35) $$M_P = G_{mP} + U_{mP} - \frac{(G_{mC} - R_m)}{q_{mP}}$$

36) $$Q_{OC} = \alpha_{OC}\,D_C$$

37) $$Q_{ON} = \alpha_{ON}\,D_N$$

38) $$Q_{OP} = \alpha_{OP}\,D_P$$

39) $$Q_N = \alpha_N\,E_N$$

40) $$Q_P = \alpha_P\,E_P$$

For the following four configurations, flux variables indicated with an * are used to replace the associated flux variables in the equations above.

Uncoupled C model:

41) $P_n^* = P_n$

42) $R_m^* = R_m$

43) $U_{nC}^* = U_{nC}$

44) $G_{mC}^* = G_{mC}$

Liebig model:

45) $U_{nC}^* = \min\left(U_{nC}, \frac{U_{nN}}{\phi_N}, \frac{U_{nP}}{\phi_P}\right)$

46) $U_{mC} = \min\begin{Bmatrix} \varepsilon_{mC} G_{mC} \\ q_{mN}\left(U_{mN} + G_{mN}\right) \\ q_{mP}\left(U_{mP} + G_{mP}\right) \end{Bmatrix}$

47) $U_{nN}^* = \phi_N U_{nC}^*$

48) $G_{mC}^* = \frac{U_{mC}}{\varepsilon_{mC}}$

49) $U_{nP}^* = \phi_P U_{nC}^*$

50) $R_m^* = G_{mC}^* - U_{mC}$

51) $P_n^* = U_{nC}^* + L_C$

52) $G_{mN}^* = G_{mC}^* \frac{r_{mN} D_N}{r_{mC} D_C}$

53) $U_{Nfix}^* = \max\left(0, U_{nN}^* - U_N + L_N\right)$

54) $G_{mP}^* = G_{mC}^* \frac{r_{mP} D_P}{r_{mC} D_C}$

55) $U_N^* = U_{nN}^* + L_N - U_{Nfix}^*$

56) $M_N^* = G_{mN}^* + U_{mN} - \frac{\left(G_{mC}^* - R_m^*\right)}{q_{mN}}$

57) $U_P^* = U_{nP}^* + L_P$

58) $M_P^* = G_{mP}^* + U_{mP} - \frac{\left(G_{mC}^* - R_m^*\right)}{q_{mP}}$

Concurrent limitation model:

59) $P_n^* = \frac{\rho_M P_n \left(U_N + U_{Nfix}\right) U_P}{B_A^2}$

60) $U_{mC} = \rho_{mM} G_{mC} \left(U_{mN} + G_{mN}\right) \times \left(U_{mP} + G_{mP}\right)$

61) $U_{nC}^* = P_n^* - L_C$

62) $G_{mC}^* = \frac{U_{mC}}{\varepsilon_{mC}}$

63) $U_{nN}^* = \phi_N U_{nC}^*$

64) $R_m^* = G_{mC}^* - U_{mC}$

65) $U_{nP}^* = \phi_P U_{nC}^*$

66) $G_{mN}^* = G_{mC}^* \frac{r_{mN} D_N}{r_{mC} D_C}$

67) $U_N^* = \max\left(0, U_{nN}^* - U_{Nfix} + L_N\right)$

68) $G_{mP}^* = G_{mC}^* \frac{r_{mP} D_P}{r_{mC} D_C}$

69) $U_{Nfix}^* = U_{nN}^* + L_N - U_N^*$

70) $M_N^* = G_{mN}^* + U_{mN} - \frac{\left(G_{mC}^* - R_m^*\right)}{q_{mN}}$

71) $U_P^* = U_{nP}^* + L_P$

72) $M_P^* = G_{mP}^* + U_{mP} - \dfrac{\left(G_{mC}^* - R_m^*\right)}{q_{mP}}$

Acclimating model:

73) $R_C = L_C \left(\dfrac{q_N B_N q_P B_P}{B_C^2} \right)^{1/4}$

74) $\eta_C = \dfrac{3 + \dfrac{\dfrac{R_m}{q_{mN}} - M_N}{U_{mN}} + \dfrac{\dfrac{R_m}{q_{mP}} - M_P}{U_{mP}}}{1 + \dfrac{\dfrac{G_{mC}}{q_{mN}} - G_{mN}}{U_{mN}} + \dfrac{\dfrac{G_{mC}}{q_{mP}} - G_{mP}}{U_{mP}}}$

75) $R_N = L_N \left(\dfrac{B_C}{q_N B_N} \right)^{1/2}$

76) $\eta_N = \eta_C \left(\dfrac{G_{mC}}{q_{mN} U_{mN}} - \dfrac{G_{mN}}{U_{mN}} \right) + \dfrac{M_N}{U_{mN}} - \dfrac{R_m}{q_{mN} U_{mN}}$

77) $R_P = L_P \left(\dfrac{B_C}{q_P B_P} \right)^{1/2}$

78) $\eta_P = \eta_C \left(\dfrac{G_{mC}}{q_{mP} U_{mP}} - \dfrac{G_{mP}}{U_{mP}} \right) + \dfrac{M_P}{U_{mP}} - \dfrac{R_m}{q_{mP} U_{mP}}$

79) $P_n^* = 3V_C P_n$

80) $G_{mC}^* = \eta_C \; G_{mC}$

81) $U_N^* = 3V_N U_N$

82) $G_{mN}^* = \eta_N \; G_{mN}$

83) $U_{Nfix}^* = 3V_N U_{Nfix}$

84) $G_{mP}^* = \eta_P \; G_{mP}$

85) $U_P^* = 3V_P U_P$

86) $U_{mN}^* = \eta_N U_{mN}$

87) $\Psi = \left(\dfrac{P_n^*}{R_C} \right)^{V_C} \left(\dfrac{U_N^* + U_{Nfix}^*}{R_N} \right)^{V_N} \left(\dfrac{U_P^*}{R_P} \right)^{V_P}$

88) $U_{mP}^* = \eta_P U_{mP}$

89) $\dfrac{dV_C}{dt} = a \; \ln\left(\Psi \dfrac{R_C}{P_n^*} \right) V_C$

90) $R_m^* = \max\left\{ \begin{matrix} \left(1 - \varepsilon_{mC}\right) G_{mC}^* \\ G_{mC}^* - q_{mN}\left(G_{mN}^* + U_{mN}^*\right) \\ G_{mC}^* - q_{mP}\left(G_{mP}^* + U_{mP}^*\right) \end{matrix} \right\}$

91) $\dfrac{dV_N}{dt} = a \; \ln\left(\Psi \dfrac{R_N}{U_N^* + U_N^*} \right) V_N$

92) $M_N^* = G_{mN}^* + U_{mN} - \dfrac{\left(G_{mC}^* - R_m^*\right)}{q_{mN}}$

93) $\dfrac{dV_P}{dt} = a \; \ln\left(\Psi \dfrac{R_P}{U_P^*} \right) V_P$

94) $M_P^* = G_{mP}^* + U_{mP} - \dfrac{\left(G_{mC}^* - R_m^*\right)}{q_{mP}}$

95) $U_{nC}^* = P_n^* - L_C$

96) $U_{nN}^* = U_N^* + U_{Nfix}^* - L_N$

95) $U_{nP}^* = U_P^* - L_P$